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1. Full name:

   Łukasz Balbus

2. The diplomas and degrees

   - 2008, Wrocław University of Technology, Doctor of Mathematical Sciences, doctoral dissertation entitled *Nash equilibria in dynamic games: existence and approximations*
   - 2004, Wrocław University of Science and Technology, Master of Mathematical Sciences, specialty statistics, master thesis entitled *Games on Markov processes.*

3. Work experience in research units

   - Since 2012, University of Zielona Gora, assistant professor
     - I am employed as an assistant professor. I continue the research on game theory and my research area is focused on mathematical economics and mathematical modeling in economic research.
     - My didactic work concerns mathematical economics, modeling in finance, time series, modeling of time series, econometrics in finance in English, introduction to financial engineering, representative methods and mathematical statistics. I conduct individual classes for students from Erasmus and I teach them game theory. I have been also a bachelors thesis supervisor. I usually supervise the topics focused on economic applications e.g. econometrics, time series and multi-criteria analysis.
   - 2008-2012, Wroclaw University of Technology, assistant
     - I was employed as a teaching and research assistant. My research area was focused on game theory and mathematical economics.
     - My lecture, classes, and laboratories concerned standard mathematical courses and numerical methods, as well as the courses of *mathematical packages* with *Matlab* and *Mathematica.*

4. Scientific achievement within the meaning of article 16 paragraph 2 of the Bill of 14 of March 2003 on scientific degrees and titles and on degrees and titles in arts (Dz. U. 2017 r. poz. 1789):

   (a) The title of the achievement:

   Theory of equilibria in models with multigenerational altruism.

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(b) The series entitled *Nash equilibria in models with multigenerational altruism* consists of the following papers:

- Balbus, Ł., Reffett, K. and Woźny, Ł. (2012 JME). Stationary Markovian equilibria in altruistic stochastic OLG models with limited commitment. Journal of Mathematical Economics, vol. 48, pp. 115-132.
  IF2012: 0.321, 5YIF2012: 0.454, MNiSW: 15p, cited by 12 (Web of Knowledge).
- Balbus, Ł., Reffett, K. and Woźny Ł., (2013 JEDC). A constructive geometrical approach to the uniqueness of Markov stationary equilibrium in stochastic games of intergenerational altruism. Journal of Economic Dynamics and Control, vol. 37 (5), s. 1019-1039.
  IF2013: 1.057, 5YIF2013: 1.347, MNiSW: 25p, cited by 9 (Web of Knowledge).
- Balbus, Ł. , Jaśkiewicz, A. and Nowak A.S. (2014 Springer) Robust Markov Perfect Equilibria in a Dynamic Choice Model with Quasi-hyperbolic Discounting. In: Haunschmied J., Veliov V., Wrzaczek S. (eds) Dynamic Games in Economics. Dynamic Modeling and Econometrics in Economics and Finance, vol. 16. Springer, Berlin, Heidelberg.
- Balbus, Ł. and Woźny Ł., (2016 DGAA). Strategic dynamic programming methods for studying short memory equilibria in a class of stochastic games with uncountable number of states. Dynamic Games and Applications, vol. 6, pp.187208. DOI: 10.1007/s13235-015-0171-1.
  IF2016: 1.647, 5YIF2014: 1.551, MNiSW 2016: 20p.
- Balbus, Ł., Jaśkiewicz, A. and Nowak, A.S. (2015 GEB). Stochastic bequest games. Games and Economic Behavior, vol. 90, pp. 247-256.
  IF2015: 0.882, 5YIF2015: 1.283, MNiSW 2015: 20p.
- Balbus, Ł., Jaśkiewicz, A. and Nowak, A.S. (2015 JOTA). Existence of stationary Markov perfect equilibria in stochastic altruistic growth economies. Journal of Optimization Theory and Applications, vol. 165(1), pp. 295-315.
  IF2015: 1.160, 5YIF2015: 1.384, MNiSW 2015: 20p.
- Balbus, Ł., Jaśkiewicz, A. and Nowak, A.S. (2016 JME). Non-paternalistic intergenerational altruism revisited. Journal of Mathematical Economics, vol. 63, pp. 27-33, ISSN: 0304-4068, MNiSW: 15p.
  IF2016: 1.165, 5YIF2014: 0.625, MNiSW 2015: 20p.
- Balbus, Ł., Reffett, K. and Woźny, Ł. (2018 JET). On uniqueness of time-consistent Markov policies for quasi-hyperbolic consumers under uncertainty. Journal of Economic Theory, vol. 176, pp. 293-310.
  IF2017: 1.204, 5YIF2014: 1.541, MNiSW 2015: 20p.
- Balbus, Ł. (2019 TMNA). Markov Perfect Equilibria in OLG models with risk sensitive agents. Topological Methods in Nonlinear Analysis, in press

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DOI: 10.12775/TMNA.2019.016.
IF2016: 1.165, 5YIF2014: 0.625, MNiSW 2015: 20p.

- Balbus, Ł.,Jaśkiewicz, A. and Nowak, A.S. (2019 DGAA). Equilibria in Altruistic Economic Growth Models. Dynamic Games and Applications, in press DOI: 10.1007/s13235-019-00305-3.
IF2017: 1.073, 5YIF2017: 1.354, MNiSW 2016: 20p.

(c) The description and purpose of the aforementioned scientific achievement (within the meaning of article 16, paragraph 2 of the Bill of 14th March 2003)

The purpose of this research is the formulation, description, and the analysis of formulas modeling the problem of successive generations exploiting a natural resource. The generation trying to maximize the available resource is usually not indifferent to the future generation. The described models have altruistic features, since the business of future generation is taken into consideration in the utility of today's generation. But the interests of the future generations can be viewed by itself in another manner than by the previous generation. Because of that the conflict of interest occurs and the aforementioned models can be modeled by game theory. The aforementioned papers focus on the following research topics:

- Time inconsistency problems in paternalistic models,
- Altruism with paternalistic features,
- Altruism with non-paternalistic features.

Time inconsistency problems in paternalistic models

In the presented papers, I consider the problem of extraction of renewable resource in the following behind periods. The decision maker selects own consumption according to the current preferences on consumption paths. The proposed models can be interpreted in two ways: the single decision maker lives through many periods, but his preferences on consumption paths vary over time (see Strotz [94], Peleg and Yaari [82], Kydland and Prescott [62]), or in each period lives a distinct generation and that has its own preferences (see Phelps and Pollak [83], Arrow [5], Kohlberg [57]). In both cases the *time inconsistency* occurs.

My first approach to the problem of *time inconsistency* and *limited commitment* was a model of intergenerational bequests. In the papers by Balbus, Reffett, Woźny (2012 JME, 2013 JEDC), Balbus, Jaśkiewicz, Nowak (2015 JMAA, 2015 GEB) [19, 10], I consider the simplest model, where the preferences are represented by the utility dependent on own consumption and the consumption of the immediate successor. A so-called *simple altruism* occurs. Similar models were studied by Kohlberg [57], Lane and Mitra [65], Leininger [66], Amir [4] and Nowak [79].

In this kind of models, the limited commitment occurs, which means that

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any generation would rather encourage the successor to consume the maximal amount of resource left by the bequest, but has no tools to commit it any behavior. This and changing preferences over time lead to *time inconsistency* and the optimal path for the present generation may not by optimal for the next generation.

In the paper with coauthors (Balbus, Reffett, and Woźny (2013,JEDC)) [20], we show that there exist a stationary *Markov Perfect Equilibrium* in a class of Lipschitz continuous functions. Under additional assumptions, we show a uniqueness of the equilibrium in a large class of Borel measurable and bounded functions. To prove it, we use Guo, Cho, Zhu [47] Theorem about fixed points of monotone operator defined on cones. Moreover, the iterations of the best response function lead us to a unique equilibrium. As a result, we have a tool for approximation of Markov Perfect Equilibrium by successive iterations. We show one example which shows that the uniqueness of Markov Perfect Equilibria is rare in such a type of models, since we relax our assumptions just a little bit and show that there exist exactly three distinct equilibria. In the present literature, the current papers the authors focus attention on the problem of existence any equilibrium, for example Kohlberg [57], Leininger [66], Lane and Leininger [64] as well as Amir [4], but there was a gap in the results by uniqueness, approximations and convergence of approximation schemes. In fact, there are a very few papers on approximation, but it concerns discounted stochastic games only (see [17, 97]), but in those papers there are restrictive assumptions: e.g. in [17] the game is symmetric, and in [97] the probability has an additive form. We obtained similar results in the paper by Balbus, Reffett, Woźny (2012,JME) [13]. Unlike in the paper from JEDC, this model has been modified a little bit because the generation chooses a consumption and the labor supply for future generations. It is ought to be noted here that the papers in JEDC and JME provide us with many another interesting results: existence of invariant distribution generated by the dynamic of economy in equilibrium, comparative statics with respect to parameter, numerical methods for computing a unique equilibrium. In particular the perfect equilibrium can be approximated by the equilibria in the finite horizon model.

In all aforementioned paper, the conclusion is that the problem of existence of Markov Perfect Equilibria is a difficult enough problem, but a uniqueness and the approachability of equilibria needs many additional assumptions. Because of that, in the next paper, we focus attention on the existence of equilibria only under the most general assumptions for proving the existence of equilibria in the models of *simple altruism*. In fact, the standard assumptions on strict concavity of temporal utility is still acceptable, but the current assumption on the transition function has been significantly relaxed. In particular, the transition function encompasses the deterministic transition (e.g. Cobb-Douglas, CES) and the non-atomic transition probability as a special instance. I present such a model in the next paper (Balbus, Jaśkiewicz, and Nowak (2015,GEB))

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[10] in which the space of capital is unbounded. A similar model was considered by Leininger [66], but he considered a deterministic production function and a bounded space of capital. In our paper, we show the existence of a stationary Markov Perfect Equilibria in the set of investment level functions which is increasing and lower semicontinuous and the set of such functions is equipped with a *weak topology*. Similar method was used by Majumdar and Sundaram [69] as well as Dutta and Sundaram [41] in stochastic games, but they considered the functions defined on bounded space of capital.

Altruism with paternalistic features

The altruism can have various dimensions and features. We say that the altruism is paternalistic if the utility of the present generation depends on own consumption and *consumptions* of some or all future generations. In turn, the altruism is non-paternalistic if the utility of present generation depends on own consumption and the *utilities* of some or all future generations. The models presented in the previous section have paternalistic features. There are other paternalistic models moving beyond the class presented before.

Although the most general model of paternalistic altruism can be found in Bernheim and Ray [31], the most popular altruism is still *quasi-hyperbolic* considered by Phelps and Pollak [83]. The utility function is a sum of temporal utilities from the consumption and from the consumption of future generations and all temporal utilities are discounted by a constant altruistic coefficient and constant discount coefficients from the current period to the next. Similar utility function can be found in many other problems like Harris and Laibson [48], Laibson [63], Karp [54], and in dynamic games, e.g. Chade, Prokopovych and Smith [35], Alj and Haurie [1], Balbus and Nowak [18], and L. Maliar and S. Maliar ([70]).

We have presented the models with quasi-hyperbolic altruism in the following papers Balbus and Woźny (2016, DGAA) and Balbus, Reffett, and Woźny (2018, JET) [27, 26]. In the first mentioned paper, we prove the existence of perfect equilibria in the class of Markov policies defined on uncountable state space. For this purpose, we apply a method in the spirit of Abreu, Pearce and Strachetti [2], and Mertens and Partasarathy [77] where the authors studied the stochastic games. In particular, we show the characterization of the set of all equilibria values, which is the intersection of descending family of compact sets with respect to *weak topology* adapted to this situation. Our method is one of many other modifications of the method by Abreu, Pearce and Stracchetti. Another method can be found in the following papers: Cole, Kocherlakota [37], Judd, Yeltekin, Conklin [53], Doraszelski, Escobar [40], Sleet, Yeltekin [92], Feng, Miao, Peralta-Alva, Santos [43]. Unquestionable advantage of the aforementioned method is that it works under many restrictive assumptions needed for using Kakutani Fixed Point Theorem. In particular, this method does not require the convexity of iterating sets. But the drawback of this method is that we consider too large a class of an equilibria set and because of that we are not able to verify many desired properties, for example whether any of these equilibria are stationary.

In the second mentioned paper on quasi-hyperbolic discounting, we have verified the problem of existence and uniqueness of stationary Markov Perfect Equilibria. In the current literature, the conditions of existence equilibria (e.g. Harris and Laibson [48], Alj and Haurie [1], Balbus and Nowak [18]) can be found. Comparing with our recent paper Balbus, Reffett, and Woźny [23], we have only the existence of the greatest and the least equilibria, in turn Laibson [63] provides the conditions for a uniqueness of equilibria in the case of the finite horizon model. The existence of *stationary equilibria* requires more restrictive assumptions than Balbus and Woźny (2015, DGAA) [27]. We allow however the value function to be unbounded from above. For this purpose, we use the idea of Rincón-Zapatero and Rodríguez-Palmero ([86, 87]) and we construct a local contraction with delay 1 whose fixed point is unique and it is a utility supported by stationary Markov Perfect Nash equilibria. Moreover, the successive iterations of this operator move us towards to the unique equilibrium value, and the convergence is uniform on any compact subset of the state space. Our results provide us a tool for numerical method for computing equilibria value. In the paper by L. Maliar and S.Maliar ([70]), it can be seen how difficult this kind of problem is. Our paper contains also other interesting problems like comparative statics, Lipschitz property with a modulus 1, and the differentiability of equilibria policy.

In the next paper with coauthors Balbus, Jaśkiewicz and Nowak (2015, JOTA) [11], we show the existence of *stationary Markov Perfect Equilibria* on uncountable state space under as general assumptions as possible. The presented model has also paternalistic features and encompasses the hyperbolic discounting as a special instance. We assume that the transition function has a non-atomic distribution. We show the existence of the stationary Markov Perfect Equilibria in the space of increasing and lower semicontinuous investment functions endowed with the *weak topology*. Our paper is closely related with Bernheim and Ray [31] who considered a deterministic transition function. But one of our examples indicates that the problem of existence of stationary equilibria is still open in their model. The point is that the operator of the best response is not continuous with the weak topology under assumptions by Bernheim and Ray. Because of that, Schauder-Tichonoff Theorem cannot be applied and because of that, we can conclude that there are many interesting open problems in this field.

Beside the existence of a stationary Markov Perfect Equilibria ,our paper provides some results on existence of a invariant distribution whenever generations use a stationary equilibrium policy. Under the most general assumptions, we show that for any equilibrium policy there is at least one invariant distribu-

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tion. Moreover, we offer the results on the order structure with respect to the first order stochastic dominance. More precisely, the set of invariant distributions is not empty and is a *chain complete* poset. In fact, in the paper by Balbus, Reffett, and Woźny (2013, JEDC) [20], we show that the set of invariant distributions is an antichain, but in that paper, we accept more restrictive assumptions on the transition probability. To prove all the aforementioned theorems, we use the results by [72] and [50].

The multigenerational stochastic game with a quasi-hyperbolic discounting can be found in the paper by Balbus, Jaśkiewicz and Nowak (2014 Springer) [9], which is one of the chapters in the monography *Dynamic Games in Economics* published by Springer. Unlike in the aforementioned models, the transition probability is unknown for the decision maker. More precisely, the transition probability depends on an unknown parameter which is never revealed. Because of that, we need to define a utility for the decision maker in such a way that that the utility is independent of this parameter. We assume that the decision makers find a policy robust against the most adverse realizations of this parameter (externality). This leads to finding the *max-min* policy for each generation and the adaptation of the idea of Gilboa and Schmeidler [45] to stochastic games. The equilibrium in this model is called a *robust equilibrium*, and it is a profile of policy that any generation maximizes its own utility, assuming that the value of this parameter has the most negative effect on own utility. We prove the existence of robust equilibrium in the class of increasing and lower semicontinuous investment functions. The presence of unknown parameters makes this problem so complex that it leads us to reconsider the transition function in a similar form as in Balbus, Reffett, and Woźny (2013 JEDC) [20].

Altruism with non-paternalistic features

The altruism with non-paternalistic features is studied by many authors in economics, for example Barro [30], Loury [68], Ray [84]. The utility for generations can be expressed by *recursive utilities* introduced by Koopmans [58], and was extended further by Kreps and Porteus [61], Kreps [60], and Epstein and Zin [42]. Epstein and Zin consider the most general model of the utility, where the consumption paths are random, and the random future utility is parametrized by the so-called *conditional certainty equivalent*. This is a generalization of the expectation, motivated by experiments by Ellsberg and Allais, which point to the fact that the preferences of players in a casino are usually far from standard expectation. The maximal utility for the present can be expressed by the generalized Bellman equations. Another certainty equivalence known in the literature are entropic risk measure postulated by Weil [102] which is a special case of the quasi-arithmetic mean introduced by Chew [36]. Other equivalents can be found in such papers as Koopmans, Diamond, and Williamson [59],

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Dekel [38], Gul [46]. Because of the *transversality* of recursive utility, the optimal consumption path for the current generation, if it exists, is consistent with the optimal politic for any future generation. Because of that, the problem of existence of equilibria is the same as the maximization of the utility for any generation and the time inconsistency does not occur. In particular, as it is shown in Saez Marti and Weibull [89], the standard Markov Decision Problem by Samuelson [90], Blackwell [33], or Strauch [93] can be viewed either as a problem of the single desision maker or the problem of many decision makers and the perfect equilibrium coincides with the optimal policy.

In the paper by Balbus, Jaśkiewicz, and Nowak (BJN 2016) [13] published in *Journal of Mathematical Economics*, we consider a model with altruism, having non-paternalistic features. We consider a model by Raya [84] in which the preferences of current generation are represented by the utility function dependent on the utilities of all future generations. Because of that, the utility is more general than that in Koopmans [58] and Epstein and Zin [42], but is more related with Galperti and Strulovici [44]. By that example, we show that the problem of existence of equilibria in Ray [84] is still open. Instead, we accept the assumption that the distribution of the next state is a non-atomic measure and we show the existence of Markov perfect equilibria.

The altruistic models with paternalistic and non-paternalistic features was studied first by Hori [51]. Hori considered a deterministic case. My next paper was publish in *Topological Methods in Nonlinear Analysis* and I studied an extension of Hori model toward a random transition function. The motivation for the research is not only the construction of the bridge between the paternalistic models (see Laibson [63], Harris and Laibson [48], Alj and Haurie [1]) and non-paternalistic models (Loury [68], Ozaki and Streufert [81], Marinacci and Montrucchio [71], Le-Van and Vailakis [67], [32] and [34]), but also the natural interpretation that the preferences of generations living for many periods combine both paternalistic and non-paternalistic features (see Doepke and Zilibotti [39]). In the presented paper, I consider rather general assumptions of the aggregator which is a contraction with respect to one of its arguments. I constructed the local contraction with delay 1 using the idea of Rincón-Zapatero and Rodríguez-Palmero [86, 87] and the results of Nowak and Matkowski [76] and also Martins-da-Rocha and Vailakis [73]. I have proven the existence of a unique recursive utility function in the class of locally bounded functions. Using the results above, I have constructed the best response correspondence and using Kakutani Fixed Point Theorem, I have shown the existence of Markov Perfect Equilibria in a class of mixed policies (see also Balbus and Nowak [18], Nowak [80] and He and Sun [49]).

Finally, I would like to present the last paper in this series, namely Balbus, Ł.,Jaśkiewicz, A. and Nowak, A.S. (2019, DGAA) [15]. Similarly as in Balbus (2019, TMNA) [6], we study a model with paternalistic and non-paternalistic features. Unlike in Balbus (2019, TMNA) [6], we show that there is a Markov

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Perfect equilibrium in the class of pure strategies, that are increasing function as investment functions on state. Unlike in the recent paper Balbus, Jaśkiewicz, Nowak (2015, GEB) [10], the aggregator does not have an additive form. This generalization is significant not only for mathematical reason, but also economic since it allows to extend our model in a wider class of economic models, e.g. we can use an aggregator in Koopmans, Diamond and Williamson [59] between the present consumption and *aggregated* consumption and utility of the next generation. But unlike in Balbus (2019, TMNA) [6], we accept additional assumptions on the aggregator, as additional aggregation between consumption of the successor and his/her utility. More precisely, the utility for the consument depends on the present consumption and the next generation by the aggregation between its consumption and its investment. Similarly as in Balbus (2019, TMNA) [6], we show the existence of a unique recursive utility in the class of bounded functions with respect to the weight-norm. On the other hand, in the proof, we apply Matkowski Theorem (see [75]) which extends Banach Contraction Principle. Next, we prove the existence of a pure equilibrium using Schauder Tychonoff Fixed Point Theorem.

5. **Other important publications**

Beside the aforementioned papers, I have published many other papers, related at different levels with these in the series of topically related publications. In my papers the dynamic supermodular games and the static supermodular games with continuum agents can be found.

The closest related paper can be found in Balbus, Reffett, and Woźny [23] published in *International Journal of Game Theory* in Balbus, Jaśkiewicz, and Nowak ([12, 14]) and in Balbus, Jaśkiewicz, Nowak, and Woźny [16] as well as [25] a survey on dynamic games in economics coauthored with K. Reffett and Ł. Woźny. The paper by Balbus, Reffett, and Woźny [23]) is a paper on the base of which we have published a further paper in *Journal of Economic Theory.* We study the problem of multigenerational supermodular games with a quasi-hyperbolic discounting. With Tarski Theorem [98], we prove the existence of the greatest and the least Markov Perfect Equilibria, which are fixed points of two possibly distinct comparable operators. Moreover, iterating successively the greatest operator, we obtain the sequence of iterations approaching the greatest equilibrium, but iterating the smallest one, we obtain the sequence of iterations approaching the least equilibrium. In the current papers, using Kakutani or Fan-Gliksberg the existence of at least one equilibria was shown, but the authors did not suggest much on the ordinal structure of equilibria (see Alj and Haurie [1], Harris and Laibson [48]). Next papers closely related with the series are the papers coauthored by Balbus, Jaśkiewicz, and Nowak ([12, 14]). Both of these papers concern paternalistic altruism, and the last mentioned extends the model with quasi-hyperbolic altruism. The paper Balbus, Jaśkiewicz, and Nowak [12] extends the results by Leininger [66] toward the case of an unbounded below utility function. To prove the existence of equilibria, we propose an

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alternative method which is to find an equilibria in the set of the lower semicontinuous endowed with a weak topology. Similar methods were used by Sundaram [96], Majumdar and Sundaram [69] and Dutta and Sundaram [41] in stochastic games. Moreover, we make precise assumptions for each selection of the best response map to be increasing in investments. Leininger assumed this property. Moreover, we have more general examples than those in Leininger. In the papers [14] and [16] we also operate on the weak topology. In [14] we study a quasi -hyperbolic discounting model and in [16] we study a bequest model. We adapt the method proposed in [27] but comparing with that model, we nowhere assume the stationary model and the capital space is unbounded.

Together with coauthors (Balbus, Reffett, and Woźny [21, 22]), we publish two papers on stochastic supermodular games. In *Journal of Economic Theory* [21], we obtained the existence of the greatest and the smallest Nash equilibria in the set of stationary policies. Both extreme equilibria can be obtained as the approach of equilibria in finite horizon models. Compared with recent paper Balbus and Nowak [18], those authors consider a symmetric game which is not supermodular. We found similar methods in *Dynamic Games and Applications* [22] in which we considered Bayesian version of supermodular game.

Together with coauthors Balbus, Dziewulski, Reffett, and Woźny [7, 8], we published two papers in *Economic Theory*. In both of them, we consider a problem of existence and approximation of distributive Nash equilibria in a sense of Mas-Colell [74] and Schmeidler [91] in a static supermodular game. In [8], we analyze a static large game with strategic complementarities. We show how the complementarities in such an environment are formally defined (in the space of strategies (see Schmeidler 1973) and in distributive games (see Mas Colell 1984)) and prove some properties of the strategy space (measurable functions) mapping the characterisics of agents into the action. Similarly for distributional games, we show how to define the complementarities on distributions and show some properties of the set of distributions with properly defined orders. In the paper we show conditions for the existence of equilibria (Nash and distributive) in both models and present the tool for computing some selected elements from the set of equilibria. Our assumptions are different than in the recent papers (see Balder [28], Ali Khan [55] and Wiszniewska-Matyszkiel [101]). Beside the existence, we prove that the extreme equilibria can be characterized by a comparative static. The results extend many recent papers on supermodular games with finite space of agents: Vives [99], Milgrom, and Roberts [78]. We finish this paper with some applications of (nonaggregative) large games in social dissonance (see Akerlof [3]), games with multiple stopping, or models with keeping up with Joneses. In the paper by Balbus, Dziewulski, Reffett, and Woźny [7], we study the model with differential information. We define the available information and complementary behavior in such conditions and we present the existence results of Bayes-Nash equilibrium, its computing and comparative statics. In such a way, we extend the results by Balder and Rustichini [29] and Kim and Yannelis [56] in

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case of supermodular games. As examples of applications, we show the explanation of rebels (*riot games*), the asset valuation pricing method in incomplete markets, or auction of common value. In this paper, we also generalize the methods of aggregation of ordinal properties like quasi-supermodularity and the single crossing property in a spirit of Quah and Strulovici [85].

Together with coauthors Balbus, Reffett, and Woźny [24] we have published a note concerning a large static game. We show that the order topology and the interval topology are not equivalent for sets having essentially greater power than *continuum* (more precisely if the interval topology is not second countable). Because of that, the family of Borel sets goes beyond the class of sigma field generated by intervals and the monotone functions need not be Borel measurables. This makes some problems in some economic models e.g. bayesian large dames. Namely, if we requires the independence of agents, we would require that the space of characteristics of agents to be *saturated* (see Sun [95]). But then its power set is strictly greater than continuum. By our results, we cannot conclude that the monotone function is Borel measurable and the complementarity assumption (see Vives and Van Zandt [100]) is useless in case of saturated large bayesian game.

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