Attachment 3A

Review of my Professional and Research Achievements

1. Introduction

The aim of this review is to present the scope of my academic work since obtaining a doctorate in mathematics in 1994 at the University of Bristol, England. Section 2 presents a summary of my academic and research career. Section 3 describes a series of ten papers with a common theme, which forms the core of my application for habilitation in the field of economics.

2. My academic and research career

2.1 Basic Information

Full name: David Mark Ramsey

Academic qualifications
I obtained the title of Doctor of Philosophy (speciality: statistics) in 1994 from the Department of Mathematics, University of Bristol, UK. Title of doctoral thesis: *Evolution, Learning and Interaction in Sequential Decision Processes*.

I obtained a Batchelor's degree in mathematics (*1st class honours*) in 1990 from the Department of Mathematics, University of Bristol, UK. Specialisation: optimisation and statistics.

Employment
10/2012- Department of Computer Science and Statistics, Wrocław University of Technology. Position: Lecturer in the Operations Research and Applications of Computer Science Group.

9/2006-9/2012 Department of Mathematics and Statistics, University of Limerick, Ireland. Position: Lecturer in the Statistics Group.

10/2000-9/2006 Faculty of Fundamental Problems in Technology, Wrocław University of Technology. Position: Lecturer in the Institute of Mathematics and Computer Science (until 2005 the Institute of Mathematics).

10/1996-9/2000 Faculty of Fundamental Problems in Technology, Wrocław University of Technology. Position: Teaching Assistant in the Institute of Mathematics.

1/1995-8/1995 Department of Mathematical Sciences, University of Bath, UK. Position: Lecturer in the Statistics Group

- 10/1995-9/1995 I received a grant from the Polish Institute of Culture to study in the Institute of Mathematics, Wrocław University of Technology.

2.2 Description of my academic and research career

I completed my undergraduate studies in mathematics at the Department of Mathematics, University of Bristol, UK in 1990. In the final years of my studies, my elective courses were in optimization and statistics, which played a highly significant role in my academic development, particularly my interest in the application of mathematical techniques to economics and biology.

In 1990 I took up my doctoral studies in the same department in the statistics group under the supervision of Prof. John McNamara and Dr. Edmund Collins. The theme of my doctoral thesis involved the theory of sequential decision processes under uncertainty and competition (Title: *Evolution, Learning and Interaction in Sequential Decision Processes*). In my thesis I made use of various techniques used in operations research, in particular dynamic programming and game theory, in order to study models based on the classical job search problem (in biology this problem is known as the mate choice problem). During this period, I taught tutorials in probability theory and statistics. I obtained my PhD in 1994. The external reviewer of my thesis was Prof. Patsy Haccou (University of Leiden, the Netherlands). Two articles, which later appeared in renowned international journals, developed some of the themes from this thesis. The first, *Learning rules for optimal selection in a varying environment: mate choice revisited* (Behavioral Ecology), developed some of the ideas from the penultimate chapter of the thesis, which considered sequential decisions under uncertainty and the second, *A large population game theoretic model of job-search with discounting* (International Game Theory Review) developed work from the final chapter of the thesis, which involved game theory. Both of these articles considered variants of the job search problem, which plays a highly prominent role in my research in the field of game theory and its applications to the field of economics.

From January to September 1995, I worked as a statistics lecturer in the Department of Mathematics at the University of Bath. I taught courses in time series analysis, biostatistics and non-parametric statistics. In the same year I obtained a 12 month grant from the Polish Cultural Institute, as a result of which I came to Poland where I began collaborating with Prof. Krzysztof Szajowski from the Institute of Mathematics at Wrocław University of Technology. This cooperation resulted in a number of articles on game theoretic models of optimal stopping, which appeared in national and international publications. The most important of these articles are: *Randomized stopping times in Dynkin Games*. (ZAMM-Journal of Applied Mathematics and Mechanics), and *Selection of a correlated equilibrium in Markov stopping games* (European Journal of Operational Research). Both of these articles concern game theoretic versions of problems based on the classical job search problem.

They consider a game in which two players ("employers") observe a sequence of candidates and must employ one. In the first article, these employers strictly compete with each other. This article illustrates the fact that even in the most simple of markets, the optimal decision of a given player may depend on the decisions of other players (in the classical model the optimal decision of the player is uniquely defined). Under such uncertainty, there exists a Nash equilibrium of the game at which an employer accepts a given candidate with a probability strictly between 0 and 1. The second of these articles considers a situation in which the employers are two departments of the same organisation. Thus, although there exists a certain level of competition between the players, they also have common interests. In such a case, the players may communicate in order to attain a so called correlated equilibrium.

I attended the seminar of Prof. Krzysztof Szajowski, where various applications of probability theory and game theory were considered, in particular to economics and environmental protection. At the turn of the century, I began attending the seminar of Prof. Witold Klonecki, where the subject of interest was statistics, in particular statistical genetics. Via this group, I established contact with Prof. Maria Sąsiadek, Director of the Genetics Institute at Wrocław Medical University. This cooperation is ongoing and has resulted in a number of articles concerning genetic oncology. Also within the framework of this seminar, I met Prof. Andreas Futschik from the University of Vienna. This resulted in me teaching lectures in statistical genetics to masters students at the University of Vienna in 2005 and 2007. This collaboration also resulted in a sequence of articles regarding the statistical analysis of data from a genome sequencer, most importantly *DNA pooling and statistical tests for the detection of single nucleotide polymorphisms* (Statistical Applications in Genetics and Molecular Biology). At around the same time, I began my own independent research on game-theoretic versions of the job search problem. The following articles were results of this work: *Correlated equilibria in n-player stopping games* (Scientiae Mathematicae Japonicae), which extended the results on correlated equilibria in stopping games to games with *n* players (employers), *A model of a 2-player stopping game with priority and asynchronous observation* (Mathematical Methods of Operations Research), which considered a model in which one of the players does not have full information regarding the value of the candidates. However, the most important article from this period was *A large population job search game with discrete time* (European Journal of Operational Research). This article was part of a series of articles on game theoretical models with a continuum of players. Such models are important in the field of economics, since markets are normally composed of a large number of players. Often no individual player has an influence on the state of the market, but the behaviour of the population as a whole has an important effect. This article considers a model of a seasonal market in which a continuum of players (job seekers) are looking for some resource (job). The distribution of the values of jobs is known. When a job seeker accepts a job, both the job seeker and the post disappear from the market. Thus, during the search process, the distribution of the values of available jobs changes according to the strategies used by the players.

Whilst working in the Institute of Mathematics (which later became the Institute of Mathematics and Computer Science) I taught classes in a wide range of subjects, including probability theory, statistics and game theory. My propositions for the themes of undergraduate projects were always popular. I was the supervisor of around 30 final year projects in mathematical computer science, mainly in the fields of game theory and statistical genetics. The following subjects were among those considered: the evolution of cooperation, the evolution of aggression, game theoretical models of competition between firms. In addition, I was the supervisor of 3 masters projects in mathematics. One of these projects developed the ideas from the earlier article *Selection of a correlated equilibrium in Markov stopping games.* This thesis assumed that the players could not only communicate, but side payments were also allowed. Such a game should be treated as a cooperative game. In order to define the solution of such a game, it is necessary to calculate an appropriate value for the game (e.g. the Shapley value) and then derive a strategy profile which will achieve this value. This masters thesis was the basis of the article *Cooperative strategies in stopping games* (Annals of the International Society of Dynamic Games).

During this period, I also wrote programs for a number of courses, including Probability Theory, Applied Statistics and Game Theory. In the middle of the first decade of the new century, I began lecturing in game theory to students of the final year of the masters degree in mathematics. These classes formed, in a certain sense, my first self-authored course, since I had a large degree of freedom in writing the program. A part of this lecture course, I considered applications of game theory to economics and political science. In 2003 I was elected to the Institute Committee as a representative of the non-habilitated academics.

In September 2006 I began working in the Department of Mathematics and Statistics at the University of Limerick in Ireland, where I taught courses in Probability Theory, Statistics and Operations Research. During my six-year stay there I was heavily involved in administrative work. In 2007 I was the co-organiser of two conferences. One of these was the national *Conference on Applied Statistics in Ireland (CASI - the annual conference of the Irish Statistical Association)* and the other was the international conference *Workshop on Correlated Data Modelling (WCDM).* From 2008 to 2012 I was director of the *Economics and Mathematical Sciences* course. In this role, I introduced very important changes in the program, which I presented and justified to the university's academic council. Earlier students of this course had not been able to take the operations research courses (methods from this field of mathematics are often applied to economics). I taught two courses in operations research. The first covered linear programming and decision theory. The second course, which I designed myself, covered decision processes and game theory. The changes in the original program were implemented to adapt the course to the needs of students of the *Economics and Mathematical Sciences* and *Financial Mathematics* degrees (e.g. the program contained applications of game theory to economics). Within the role of course director, I was the chair of the course board, which was made up of lecturers from the institutes of mathematics and economics, as well as students. I also gave advice to students regarding their studies and future career. From 2008 to 2012 I was the representative of the Science & Engineering Faculty on the Business Faculty Committee.

Thanks to the new contacts resulting from the work described above, I extended my interdisciplinary research, particularly in the field of economics. In this period, I collaborated with Dr. Stephen Kinsella, from the Economics Department. This cooperation resulted in two articles: *A Model of Partnership Formation with Friction and Multiple Criteria* (in Search Theory: A Game Theoretic Perspective) and *The Optimal Incidence of Taxation in Coupled Markets* (presented at the „Irish Economic Association Annual Conference"). The first of these articles belongs to a cycle of articles on game-theoretical models based on the job search problem. The most important article from this period, of which I was the only author, is *Partnership formation based on multiple traits* (European Journal of Operational Research). This article considers a game in which the players (employers and job seekers) first obtain a signal from a potential partner (e.g. this could be a job seeker's application for a given position). Based on this signal, a player must decide whether he wishes to take part in an interview or not. After such an interview a pair made up of an employer and a job seeker must decide whether to become partners or not (i.e. whether the employer hires the job seeker or not). The article *Mutual mate choice with multiple criteria* (Annals of the International Society of Dynamic Games) considers a similar game (in this article the traits of the players are discrete and the equilibrium is derived using an algorithm rather than by solving the appropriate differential equation). At this time, I started working with Prof. Steve Alpern (who at that time was working in the Operations Research Group at London School of Economics and is presently Professor of Operations Research at Warwick Business School). This collaboration resulted in several articles on large population job search/mate choice games, which were novel due to the fact that the traits of players were not fixed, but changed over time according to an appropriate process. This takes into account the fact that the value of a job seeker may change over time, since he can gain experience or learn new skills. The most important of these articles is *Partnership formation with age-dependent preferences* (European Journal of Operational Research). Other articles in this set are *Equilibrium population dynamics when mating is by mutual choice based on age* (Theoretical Population Biology), as well as my single author paper *Some generalizations of a mutual mate choice problem with age preferences* (Annals of the International Society of Dynamic Games). At the same time, I worked on other large population games. The article *Large population evolutionary games played within a life history framework* (Operations Research and Decisions) considers a model of such a game in which a player passes through various states with an intensity that depends on the strategy he uses. In certain states, he meets other players and obtains a payoff which depends on the profile of the strategies used by these players. In the simplest version of this game, neither a player's lifespan (how long he spends in a game) nor the traits of the player depend on his strategy. In such a case a player should maximize the intensity of payoffs. The article *Strategy dependent mortality in life history games* (Proceedings of the 2009 International Conference on Game Theory for Networks) considers a game in which a player's lifespan depends on his strategy. In this case a player should maximize his (possibly discounted) sum of payoffs. The article *A large population parental care game: polymorphisms and feedback between patterns of care and the operational sex ratio* (Journal of Theoretical Biology) presents an application of this theory to evolutionary biology.

From 2007 to 2012 I was a member of the „BIO-SI" research group, which was based around four academic researchers, two from the University of Limerick (the other was Prof. Gilbert MacKenzie) and two from the National University of Ireland, Galway (Prof. John Hinde and Dr. John Newell). In 2008 this group obtained a grant of €500 000 from „Science Foundation Ireland" in order to develop statistical models in biology and medicine. Within the framework of this project, I was the supervisor of the doctoral thesis of Ali Sheikhi Mehrabadi entitled *„Statistical Methods for the Detection of Single Nucleotide Polymorphisms (SNPs) Using New Generation Genome Sequencers".* Ali Sheikhi Mehrabadi passed his PhD viva in February 2014. In addition, I was the main organiser of the conference *Symposium on Biostatistics and Statistical Genetics,* which took part in Limerick in September 2009.

The MACSI research group *(Mathematical Consortium for Science and Industry)* was also very active in the Department of Mathematics and Statistics. This group was funded by *„Science Foundation Ireland",* to cooperate with industry and researchers from other fields. Within the framework of this research group, I was one of the supervisors (together with Prof. James Gleeson, who was the main supervisor and Dr. John Kinsella) of the doctoral thesis of Dr. Mel Devine. This research was carried with the cooperation of the firm "Bord Gais". Some of the results of this thesis were published in the article *A Rolling Optimisation Model of the UK Natural Gas Market,* which appeared in the journal „Networks & Spatial Economics". This article considers the problem of a firm which wants to derive an optimal strategy for supplying gas to its clients. Above all, it is necessary to decide when and where to buy the gas (when demand is lower in spring and summer, it is possible to buy gas at a lower price and store it in order to satisfy the higher demand in winter, but storage is also associated with costs). This article first describes a model describing the demand for gas in the UK and derive an optimal strategy on the basis of this model. In 2011 I also became the supervisor of the doctoral thesis of Ana Maria Magdalina entitled *A Bayesian Approach to Regime Changes in Multiple Linear Regression*. This thesis applies Markov Chain Monte Carlo (MCMC) methods to investigate where or when changes in a physical or economic process occur. In addition, I was the supervisor of one masters project written by Muriel Tronc, a student of Mathematical Computer Science from the Erasmus program. Her thesis considered algorithms for deriving the equilibrium of "job search" type games. I was also the supervisor of approx. 10 final year undergraduate projects of students mostly from the *Financial Mathematics* and *Economics and Mathematical Sciences* courses. The following were among the themes of these projects: competition and collaboration, game theory and climate change, and weather derivatives.

In July 2007 I took part in the *European Conference on Operational Research* in Prague, where I met Prof. Jacek Mercik (from the Department of Computer Science and Management, my present place of employment). That meeting was the beginning of our co-operation. As a result, I was one of the organisers of the *SING4 Conference on Game Theory,* which took place in Wrocław in June 2008. In 2010 I became an editor of the journal *Operations Research and Decisions* published by Wrocław University of Technology. In May 2011 I gave lectures and tutorials in probability theory (in English) as a visiting lecturer in the Department of Computer Science and

Management. In 2012 I became an editor of two other national journals: *Matematyka Stosowana (Mathematica Applicanda)* and *Economic and Environmental Studies.*

In October 2012, I began working as a lecturer in the Operations Research and Computer Applications Group in the Department of Computer Science and Management at Wrocław University of Technology. I give lectures and computer laboratories in statistics, econometrics and probability theory in both Polish and English.

In 2013 I took part in the *26th European Conference on Operational Research,* where I was the chair of a session devoted to search games. In July and August 2013 I obtained a grant from the Netherlands Ministry of Education, Culture and Science to work with Prof. Robbert Fokkink on search games at the University of Delft. At the same time, Dr. Urszula Markowska-Przybyła, from Wrocław University of Economics, and I have formed a research team which obtained a grant of 300 000PLN from the Polish Ministry of Science and Higher Education in order to study the level of generalized trust among Polish students using experimental game theory. These experiments will be carried out this year. Generalized trust is a very important component of social capital, which is crucial to building an effective free market. We have just submitted the first article resulting from this project: *A Game Theoretical Study of Generalized Trust and Reciprocation in Poland: I. Theory and Experimental Design,* to Operations Research and Decisions. In 2013 I took part in three national economics conferences: Innovations for and Transitions to Sustainable Transport Systems, *12th April, WSB School of Banking in Wrocław,* where I chaired a session, *Conference on Sustainable Business and Transitions for Sustainable Development,* 20-22 June, University of Opole, where I gave a talk entitled *Land Transport Policy in Ireland and Poland: A Game Theorist's View* (an extended version of this talk will appear in *Economic and Environmental Studies*), as well as *Aktualne Aspekty Polityki Społeczno-Gospodarczej i Przestrzennej*, 23-25 September, Jelenia Góra-Cieplice, where I gave a talk entitled *Waloryzacja nieruchomości mieszkalnej na wtórnym rynku we Wrocławiu* (Valorisation of residential real estate on the secondary market in Wrocław) (the article based on this talk will appear in the journal *Prace Naukowe UE we Wrocławiu*). These facts indicate that I am known and respected internationally in the field of game theory and an active participant in academic life in Poland.

3. Review of a series of 10 papers with a common theme of which I was author or co-author

The centre piece of my application for habilitation is a cycle of ten papers with a common theme in the field of dynamic games, in particular models based on the classical job search problem. Nine of these articles appeared in peer reviewed journals, one in a Polish journal and the remainder in renowned international journals (six on the ISI list). The remaining article appeared as a peer reviewed chapter of a book published by the international publisher *Springer*. The following articles constitute the aforementioned series of articles (in the order in which they are described in the review)[1]:

1. Collins, E.J., McNamara, J.M., Ramsey, D.M. (2006) *Learning rules for optimal selection in a varying environment: mate choice revisited.* Behavioral Ecology, 17(5): 799-809.

2. Neumann, P., Ramsey, D., Szajowski, K. (2001) *Randomized stopping times in Dynkin Games*. ZAMM-Journal of Applied Mathematics and Mechanics, 82: 811-819.

3. Ramsey, D.M. (2007) *A model of a 2-player stopping game with priority and asynchronous observation*. Mathematical Methods of Operations Research, 66(1): 149-164.

4. Ramsey, D.M., Szajowski, K. (2008) *Selection of a correlated equilibrium in Markov stopping games.* European Journal of Operational Research, **184:** 185-206.

5. Ramsey, D.M. (2008) *A large population job search game with discrete time.* European Journal of Operational Research, **188(2)**: 586-602.

6. Ramsey, D.M. (2009) *A large population game theoretic model of job-search with discounting.* International Game Theory Review, 11(3): 301-320.

7. Ramsey, D.M. (2012) *Partnership formation based on multiple traits.* European Journal of Operational Research, **216:** 624-637.

8. Kinsella S., Ramsey D.M. (2013) A Model of Partnership Formation with Friction and Multiple Criteria. In: Alpern S., Fokkink R., Gąsieniec L., Lindelauf R., Subrahmanian V.S. (eds.), *Search Theory: A Game Theoretic Perspective*, s. 267-293. Springer Science & Business, New York.

9. Alpern, S., Katrantzi, I., Ramsey, D.M. (2013) *Partnership formation with age-dependent preferences.* European Journal of Operational Research, **225:** 91-99.

10. Ramsey D. M. (2009) *Large population evolutionary games played within a life history framework.* Operations Research and Decisions **7(2)**: 51-74.

All of these articles use the theory of sequential decision processes and, apart from the first one, present models of dynamic games. They model the search for a particular resource. This approach is used in economics to model e.g. the employment market.

Stigler G. J. (1961) was the first author to consider a model of this type. He assumed that a client is looking for a non-differentiated good, which is available in a large number of shops. The client does not have any information regarding the price in a given shop, but knows the distribution of prices. He visits the shops in a random order (i.e. the prices observed are independent realizations from the distribution of the prices) and on visiting a shop must decide whether to buy the good at the given price (and hence stop searching) or continue searching. The goal of the client is to obtain the good

[1] Some of these articles were written with the cooperation of co-authors. A description of my role in these papers is described in Attachment 6.

at the lowest cost, where the cost is defined to be the sum of the price of the good and the search costs (assumed to be proportional to the number of shops visited). This model leads to a problem of optimal stopping, where under the optimal strategy, the client buys the good in the first shop where the price is lower than an appropriately defined threshold. This threshold is equal to the expected costs from future search. According to such a model, the fact that there exists various prices on the market results from the fact that clients have different search costs.

In the field of economics, a model of this type is often used to describe the employment market (job search models). In the classical model of this type, a job seeker observes a sequence of job offers with values from a known distribution and employers are not choosy. When a job seeker wants to accept an offer, he is employed and stops searching. This classical model leads to an optimisation problem in which the distribution of the values of jobs does not depend on the behaviour of other players and is known to all job searchers. In biology, an analogous model is used in mate choice theory in the case where only one of the sexes is choosy. The articles in the series described in this review have weaker assumptions.

The first article *Learning rules for optimal selection in a varying environment: mate choice revisited* considers a model, in which (using the language of the job search problem) job seekers do not know the distribution of the value of jobs, which does not change over time, and collect information regarding this distribution during the search process. It is assumed that a job seeker may observe an unlimited number of offers. Using a Bayesian approach to such a problem, under the optimal strategy a job seeker should accept a job whose value exceeds an appropriately defined threshold which depends on his current estimate of the mean value of jobs and decreases to an asymptotic value as the search process proceeds. It follows from this that an optimally behaving job seeker who has already rejected a large number of jobs, and thus knows the distribution of job values, should use a fixed threshold. The form of this strategy results from the need to balance the need for information regarding the distribution of the value of jobs with the costs incurred when a good job is rejected. Under the assumptions of the classical model, the opportunity of returning to a previously rejected job has no influence on the form of the optimal strategy. This results from the fact that, according to the classical model, the optimal strategy is defined by a fixed threshold, so if a job was rejected in the past it remains unacceptable. When the distribution of the values of jobs is unknown, the possibility of returning to a previous offer has an influence on the optimal strategy, since under the optimal strategy the threshold used changes over time. For example, assume that a job seeker initially thinks that the job market is more favourable that it really is. In this case, if the first job observed (of value x) is relatively good, the job seeker may well reject it. However, after observing several worse offers, he would accept a job of value x. In this article, we consider a model in which there is no possibility of return to a previous offer. In order to derive the optimal strategy using a Bayesian approach, we assume that the job seeker should have an appropriate *a priori* estimate of the mean value of jobs and an estimate of the variance of such an estimator with respect to the variance of the values of jobs. In practice, such prior estimates may be inappropriate. Hence, this article considers the effectiveness of non-learning strategies (strategies defined by fixed thresholds) and simple learning strategies (these strategies use a threshold relative to the searcher's present estimate of the mean value of jobs). Learning strategies are very effective when there exists a large degree of uncertainty regarding the mean value of jobs and the variance of the value of jobs is large. In this case, strategies which initially overestimate the mean value of jobs and use a fixed threshold relative to the current estimate of the mean value of jobs have very similar properties to the optimal Bayesian decision rule (under such a strategy, the initial estimate of the mean value of jobs is greater than under the optimal Bayes procedure, but decreases more quickly when the job seeker does not find an acceptable job). If we assume that a young job seeker uses the mean value of all jobs on the market as an estimate of the distribution of the values of jobs available to him and that the probability of an employer accepting him is decreasing in the value of a job, then such a strategy may describe the behaviour of such a job seeker well. This results from the fact that such a job seeker will rapidly lower his estimate of the mean value of jobs available to him.

In reality, job seekers (and employers) compete with each other and both types of player are choosy. Hence, the values of the job seekers and employers on the employment market depend on the strategies used by both job seekers and employers. Thus it is possible to consider the job search problem as a game in which the players belong to a given population, which consists of two types of player (employers and job seekers). Each player has a given value. From the point of view of an employer, he observes a sequence of job seekers and employs the first for whom there is mutual acceptance, when both the employer (and the job seeker) stops searching. The payoff of the employer is equal to the value of the job seeker employed minus the search costs incurred by the employer. The problem faced by a job seeker is analogous. His payoff is equal to the value of the job (employer) obtained minus his search costs. A solution of such a game is given by the concept of a Nash equilibrium. At such an equilibrium, no player can increase their expected reward from search by unilaterally changing his strategy.

It should be noted that we assume that the players act independently of each other (i.e. there is no communication). There can be multiple equilibria of a game. For example, consider the following simple game in which there are only two employers (one good and one bad) and two job seekers (one good and one bad). We assume that the search costs are small and both employers prefer the good job seeker to the bad one and the bad job seeker to employing no one. Similarly, both job seekers prefer the good employer to the bad employer and the bad employer to being unemployed. The following strategy profile is a Nash equilibrium: i) the good employer only accepts the bad job seeker, ii) the bad employer only accepts the good job seeker, iii) the good job seeker only accepts offers from the bad employer, and iv) the bad job seeker only accepts offers from the good employer. It should be noted that at such an equilibrium the good employer hires the bad job seeker and the bad employer hires the good job seeker. Hence, the bad employer and the bad job seeker obtain their maximum possible payoff. The good employer would gain a greater payoff if he hired the good job seeker. However, given only a unilateral change in strategy, the good job seeker would not accept him. Similarly, the good job seeker would receive a greater payoff if he could work for the good employer, but given just a unilateral change in strategy, the good employer would not hire him. In reality, such an equilibrium is not economically rational, since any employer would be willing to employ the best candidate for a job. In order for an equilibrium to satisfy this condition, we require that each player accepts a potential partner whose value exceeds the expected reward of that player from future search (not taking into account the costs that have already been incurred). In their classical game theoretical job search problem in which both employers and job seekers are choosy, Collins and McNamara [1990] call this condition the „optimality criterion".

The following three articles concern models of games in which there are two employers both observing a sequence of job seekers. These models show that even in the simplest markets there can be multiple equilibria. It is assumed that the job seekers are not choosy, i.e. they accept any offer of work. The goal of the article *Randomized stopping times in Dynkin Games* is to derive the possible equilibria when the two employers strictly compete with each other. They jointly observe a sequence of candidates for some position, e.g. computer scientist, whose values (given the values of the previous candidates) have a known distribution. When only one employer wishes to accept a candidate, then that employer hires him and stops searching. In this case, the second employer can continue observing the candidates until he hires one. When both of the employers accept a candidate, then it is assumed that the first employer hires him with probability p (and the second employer continues searching). Otherwise, (i.e. with probability $1-p$) the second employer hires the candidate and the first continues searching. It is assumed that neither of the employers has absolute priority (i.e. $0 < p < 1$). In such games, when the employers observe a very good candidate, they both want to hire him (since independently of the decision of the other employer, it pays to accept such a candidate), and when the employers observe a poor candidate, neither of them wishes to hire him (since independently of the decision of the other employer, it pays to reject such a job seeker). There exists uncertainty regarding an employer's optimal decision when a relatively good candidate appears. In this case, either of the employers would prefer the other to employ such a candidate, so that he can continue searching without any further competition. However, the worst situation occurs when both employers reject such a candidate. In such situations there are three Nash equilibria, two pure equilibria where one of the employers accepts such a candidate and the other does not, and a third equilibrium at which both employers accept a candidate with the appropriate probability. The form of this mixed equilibrium is derived, together with the asymptotic form of such an equilibrium when the goal of both employers is to employ the best of all the candidates (this is a game theoretic version of the well-known secretary problem).

In the article *A model of a 2-player stopping game with priority and asynchronous observation* I consider a similar game in which one of the employers always has priority, i.e. he always observes a candidate before the other employer sees him. Hence, the form of this game is similar to Stackelberg's model of imperfect competition. It follows from this that the second employer may have incomplete information regarding the values of the previous candidates. The form of the equilibrium of such a game is derived and two examples are used to show how the informational structure of such a game influences the behaviour of these employers.

The goal of the article *Selection of a correlated equilibrium in Markov stopping games* is to describe the equilibria of a game in which the two employers are different departments of the same organisation and communication between them is possible. In this case, although there exists a certain level of competition between these departments, they also have common interests. Given than neither of these departments has automatic priority (i.e. $0 < p < 1$), conflict can exist when a relatively good, but not outstanding, candidate appears. As before, in this case both of the departments would prefer the other to hire such a candidate. However, the worst situation for both departments (and the organisation as a whole) occurs when neither of the departments hires such a candidate. In such a case, the sum of the expected payoffs at the mixed equilibrium is lower than the sum of the payoffs at either of the two pure equilibria. Without communication there is no natural way of deciding which department should accept such a candidate. In such a situation, the concept of correlated equilibrium can be used to define a solution of the game. At such an equilibrium it does not pay either of the employers to change their decision based on the agreement resulting from communication. Hence, the fundamental difference between a correlated equilibrium and a Nash equilibrium lies in the fact that the actions of the players at a correlated equilibrium can depend on each other via the communication between the players. At a Nash equilibrium, the players make their decisions independently of each other. Since the set of correlated equilibria contains the set of Nash equilibria, the problem of the choice of an appropriate equilibrium still remains. However, since they players can communicate, they may choose the appropriate equilibrium together with the aid of an appropriate criterion. In this article, we consider various criteria that can be used in order to choose such an equilibrium. For example, using the *utilitarian* criterion the players act in such a way as to maximize the sum of their expected payoffs, and using the *egalitarian* criterion they act in such a way as to maximize the minimum expected reward of the two players. In order to derive such an equilibrium, it is necessary to solve a sequence of linear programming problems. In addition, we consider two possible scenarios. In the first, the employers may only communicate at the beginning of the game (i.e. before any of the candidates appears). In the second, the employers may communicate at any point in the game (in particular, when a given candidate appears).

The following two papers take into account the fact that there are a large number of job seekers and jobs on the employment market. The article *A large population job search game with discrete time* considers a model in which the employment market is seasonal. It is assumed that there is a continuum of job seekers and positions. Employers are not choosy. New positions only appear at the beginning of the season and the distribution of their values is known. Positions are available at moments $1,2,...,n$. When the number of job seekers does not exceed the number of positions, at moment i, $1 \leq i \leq n$, each job seeker observes a position whose value comes from the distribution of values of unoccupied jobs. When a job seeker accepts a job, both the job seeker and the position leave the employment market. In this way, the distribution of the values of available jobs changes over time according to the strategies used by the job seekers.

In the classical job search problem, the distribution of the values of jobs does not change over time. It follows that we can derive the optimal strategy of a job seeker using dynamic programming. Using this approach, we find the appropriate threshold (the minimal value of an acceptable job) at each moment by recursion. At moment n, the job seeker should accept any job (since there will be no more offers). At moment *i*, $i = 1,2,...,n-1$, a job seeker should accept a job if and only if its value exceeds his expected reward from future search. This expected reward does not depend on the history of the process. When the distribution of the values of jobs varies over time, this optimality criterion still holds, i.e. a job seeker should accept a job at moment *i* if its value exceeds his expected reward from future search. On the other hand, we need to know the strategies used by job seekers in the previous moments, in order to derive the distribution of the values of currently available jobs. Hence, we cannot derive the equilibrium of such a game by recursion. We can describe such an equilibrium by the set of thresholds $(v_1,v_2,...,v_{n-1})$, where v_i is the minimal value of an acceptable job at moment *i* (at moment *n* any job should be accepted). The distribution of the values of jobs available at moment *i* depends on the thresholds $v_1,v_2,...,v_{i-1}$. This vector satisfies a system of $n-1$ equations that result from the optimality criterion. When the distribution of the values of jobs is continuous, this system of equations can be solved using numerical methods. The article also presents two approaches to deriving the equilibrium when the distribution of job values is discrete and conditions under which there exists a unique equilibrium.

In the article *A large population game theoretic model of job-search with discounting* I consider a similar game in which time is continuous and job seekers observe offers according to an appropriately defined Poisson process. Employers are not choosy. The values of jobs have a discrete distribution. When a job seeker accepts a job, both the job and the job seeker leave the employment market. Thus the distribution of the values of jobs changes over time according to the strategies used in the population. The model assumes that the time available to job seekers is unbounded, but the approach can be easily adapted to problems in which there is a finite time horizon. It is shown that a unique equilibrium exists. This equilibrium is derived using an iterative procedure. First we derive the distribution of the values of available jobs at each moment in time under the assumption that each job seeker uses the strategy π_0. Using this, we can calculate the optimal response of an individual job seeker, denoted π_1, against this background. In an analogous way, we can derive the optimal response of an individual job seeker, π_{i+1}, when the population of job seekers use the strategy π_i, $i = 1,2,...$ The limit of such a sequence of optimal responses is the Nash equilibrium of the game.

The following three articles consider more realistic models of job search. In all of these models, both employers and job seekers are choosy. The articles *Partnership formation based on multiple traits* and *A Model of Partnership Formation with Friction and Multiple Criteria* take into account the fact that e.g. employers gather information on the value of a job seeker by observing various traits and some traits are easier to observe than others. In classical models of job search the preferences of players are “common”, i.e. all employers ascribe the same value to a given job seeker. However, according to these two models, different employers can ascribe different values to the same job seeker, although e.g. all employers might prefer highly qualified workers (where qualifications are one of the traits of workers). These traits have a known joint distribution. In the article *Partnership formation based on multiple traits* these traits are continuous variables and in the article *A Model of Partnership Formation with*

Friction and Multiple Criteria discrete variables. Analogously, different job seekers may ascribe different values to a given job seeker, although e.g. all job seekers prefer higher wages to lower wages (where wages are one of the traits of a job). Time is discrete and the time available to searchers is unbounded. From the point of an employer, he observes a sequence of job seekers until he employs one (which requires mutual acceptance). At each moment, an employer first observes a signal from a randomly chosen job seeker (this signal depends on the set of traits he possesses) and on the basis of this signal the employer must decide whether to invite the job seeker to an interview (e.g. this signal could be interpreted as the qualifications of the job seeker). If both parties agree, then an interview takes place. On the basis of this interview, the employer gains more information on the value of the job seeker (e.g. this information might be interpreted as the degree to which an employer and the job seeker suit each other). After the interview, both parties must decide whether they want to work together (mutual acceptance is required) or continue searching. The payoff of the employer is defined to be the value of the job seeker employed (which depends on the traits of both the employer and the job seeker) minus the search costs, which are defined to be the sum of the interview costs (only incurred when an interview takes place) and the costs of observing the initial signals (which are proportional to the total number of job seekers observed). Job seekers face an analogous problem. Initially, a job seeker observes an initial signal regarding the value of a job (e.g. this signal can be interpreted as the wages) and if an interview takes place, then the job seeker observes how well the employer suits him.

According to the model of McNamara and Collins [1990], at equilibrium both employers and job seekers are split into k groups, denoted 1, 2,..., k, such that employers from group j only employ job seekers from group j (equivalently job seekers from group j only work for employers from group j). Such an equilibrium is called a „block separating equilibrium”. This is an equilibrium which satisfies the following condition: when at equilibrium employer A would hire job seekers C and D and employer B would hire job seeker C (in each case with mutual acceptance), then employer B would always hire job seeker D (with mutual acceptance). In both of the articles considered here, it is shown that there is a unique equilibrium satisfying the optimality criterion (although in another article I showed that multiple equilibria can occur in a game with a similar structure). When the costs of interviews are high, then the equilibrium is a „block separating equilibrium”, where interviews are not used and employers simply look at the qualifications of a candidate and a job seeker just looks at the salary. However, when the interview costs are sufficiently small, then the form of the equilibrium is essentially different. The derivation of the equilibrium depends on the type of joint distribution the traits have. When this distribution is continuous, the equilibrium can be found using the appropriate differential equation. When the distribution is discrete, the equilibrium is derived using the appropriate algorithm.

These models assume that when a player leaves the market (because he employs a job seeker or finds employment, as appropriate), then a “clone” enters the market (i.e. a player of the same class and traits). It follows from this that the joint distribution of traits does not change over time. In practice, the traits of the players entering the market do not depend on the traits of those leaving the market. Burdett and Coles [1999] consider such a model according to which new employers and job seekers enter the market at a constant rate. Such a model can be described as follows: it is assumed that the populations of jobs and job seekers are large and that jobs (employers) and job seekers enter the market with the same (constant) intensity α (i.e. the number of jobs equals the number of job seekers). The distribution of the values of job seekers entering the market, denoted F_w, (and analogously, the distribution of the values of jobs entering the market, denoted F_e) do not change in time. Each employer observes a sequence of job seekers, who appear according to a Poisson process. The intensity of this process is given by a function γ which is non-decreasing in the number of job seekers presently on the employment market. When an employer hires a job seeker (mutual acceptance is required), he stops searching and obtains a reward equal to the value of the job seeker employed, discounted according to the time spent searching. From the point of view of a job seeker, he observes a sequence of jobs, which appear according to a Poisson process of intensity γ. When a job seeker is hired, he stops searching and obtains a reward equal to the value of the job, discounted according to the time spent searching.

The distributions of the values of job seekers and jobs currently on the job market are denoted by G_w and G_e, respectively and depend on the following parameters: α, F_w and F_e, which define how quickly jobs and job seekers of a particular value appear on the employment market, as well as γ and the strategies used by employers and job seekers, which define how quickly jobs and job seekers of a particular value disappear from the employment market. It can be shown that when the parameters above are given, the distributions G_w and G_e tend to a stationary distribution. It follows from this that at equilibrium both job seekers and employers use a threshold strategy (i.e. any equilibrium is of the same form as in the game of McNamara and Collins [1990]). However, it is difficult to derive the precise form of the equilibrium due to the feedback between the distributions G_w and G_e and the strategies used by the players. In general, one may approximate such an equilibrium using an iterative method.

Burdett and Coles show that multiple equilibria satisfying the optimality criterion may exist in such a game. For example, assume that there are two types of both employer and job seeker, called bad and good, the intensity of finding candidates for the role of partner does not depend on the number of players currently on the job market and the proportion of players entering the job market that are good (both employers and job seekers) equals p, where $p < 0.5$. In this case, there can be two equilibria. At the first equilibrium, good players only accept good partners, so bad employers hire only bad job seekers. At such an equilibrium, good players spend a longer time on the job market than bad players, thus the proportion of players currently on the employment market that are good is greater than p. This means that it is worthwhile for a good employer to wait until a good job seeker appears. At the second equilibrium, none of the players are choosy (i.e. employers hire the first job seeker they see and job seekers accept the first job offer they obtain). In this case, good players make up a proportion p of those currently on the employment market. This means that it does not pay a good employer to wait until a good job seeker appears.

Thus Burdett and Coles model a dynamic employment market. Another trait of such a market (which Burdett and Coles do not take into account) is the fact that the value of a job seeker may change over time, since e.g. he can undergo a training course.

In classical job search models, the value of a player is fixed and does not change over time. Little work has been done on models in which the values of players changes over time according to some process. In the article *Partnership formation with age-dependent preferences* we consider a model in which new players appear and their values as partners change over time (according to the age of a player). According to this model, employers and job seekers appear on the job market (at age 0) at a constant rate. The age of a player is defined to be the time for which he has been on the employment market. We consider a symmetric game in which employers and job seekers appear on the job market at the same rate and can stay on the employment market for the same length of time (in another article I consider asymmetric games of the same type). Without any loss of generality, it may be assumed that players can spend one unit of time on the employment market. Employers observe a sequence of job seekers who appear according to a Poisson process with constant intensity γ. Each job seeker is chosen at random from the set of all those currently unemployed, i.e. his age comes from the distribution of the age of such workers. When an employer hires a job seeker, both leave the employment market and obtain a reward which depends on the age of both players. If an employer reaches the age of 1 without employing anybody, he leaves the employment market. The problem faced by a job seeker is analogous. In this way, the distributions of the ages of job seekers and employers depend on the strategies used by the population as a whole. It should be noted that the density of the age of job seekers at the point t, $g_w(t)$, $0 \leq t \leq 1$, is proportional to the probability that a job seeker is still looking for a job at age t (it suffices to normalize this function so that the area under the density curve is equal to 1). We show that when the payoffs of the players satisfy certain conditions, then at equilibrium a player of age t accepts a prospective partner of age $\leq f(t)$, where f is an increasing function. Hence, the equilibrium profile is given by a threshold strategy.

In order to derive the equilibrium of such a game, we use an iterative procedure. First, we assume that employers do not accept any job seekers and job seekers would accept any employer. Against such a background, the distribution of the age of job seekers is uniform. We can thus calculate the optimal strategy of an individual employer, given by the threshold $f_e(t)$, and the probability that he is still searching at age t, $p_e(t)$. Then we assume that all employers use the strategy $f_e(t)$ and the distribution of the age of employers corresponds to the function $p_e(t)$. We then calculate the optimal strategy of an individual job seeker, $f_w(t)$, and the probability that he is still searching at age t, $p_w(t)$. Further, we assume that all job seekers use the strategy $f_w(t)$ and the distribution of the age of job seekers corresponds to the function $p_w(t)$ and we calculate the optimal strategy of an individual worker against that background. This process is repeated until the functions $f_e(t)$, $p_e(t)$, $f_w(t)$ and $p_w(t)$ converge. It should be noted that when all the employers use the strategy $f_e(t)$, in general, the probability that a given employer is still searching differs from $p_e(t)$, the probability that an optimally behaving employer at the previous step is still searching. However, when the algorithm converges, the strategy of an optimally behaving employer converges to the strategy used by the other employers. It follows that if this process is convergent, then $f_e(t)$ and $f_w(t)$ give the equilibrium strategies of employers and job seekers, whereas $p_e(t)$ and $p_w(t)$ correspond to the equilibrium distribution of the ages of employers and job seekers on the employment market. It is possible to start this iterative process from a different strategy profile. Numerical results suggest that there exists a unique equilibrium of such a game. This is a symmetric equilibrium, i.e. employers use the same threshold rule as job seekers.

The final article in this series, *Large population evolutionary games played within a life history framework,* is not directly connected to the job search problem, but I plan to use the ideas contained in this article to further develop models of the employment market. In this article, I consider a game with a continuum of players, where the players may be of different classes (e.g. employers and job seekers). The players pass through various states at rates which depend on their class, strategy and the background of the game, which depends on the set of the strategies used by the other players. The payoff of a player in a given period of time depends on the states that he passed through, the time spent in each state and the strategies used by the players he encountered. In such games, when the time spent in the game and a player's traits are independent of the strategy used, then he should maximize his payoff rate. Otherwise, a player should maximize his expected sum of discounted payoffs.

For example, within the framework of such a model we could consider a population of workers. At any moment in time a worker could be studying/on a training course, working or unemployed. When a worker is studying, then he will increase his value on the employment market, but during that period he will not earn a lot. When he is unemployed he could be looking for an appropriate job or training course. Hence, in a small period of time with some probability he becomes a student or is employed (i.e. changes his state). When he is working, his payoff rate depends on his employer. It would be interesting to analyse the dynamics of such a system when job seekers and employers use appropriate strategies and see what profiles are most robust to perturbations to the system.

Additional Literature

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